NC STATE UNIVERSITY

Upcoming Operations Research PhD Final Exam

Yipeng Yang

Path Dependent Stochastic Models and Their Applications In Finance and Communications

Advisor: Dr. Tao Pang
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Abstract:

We studied the path dependent stochastic models and their applications to finance and wireless communication systems. Due to the difficulty of finding the analytical solutions of such problems, the numerical approach is the best way so far to apply. We first concerned the controlled Markov Chain approximation to solve stochastic control problem with time delay. So far little is done about the implementation of this numerical method in stochastic control problem with time delay. This method is applied to a wireless communication queuing system. The system state follows a stochastic delay differential equation (SDDE), which is derived from a heavy traffic model. Inspired by the separation principle for control systems with partial information, we added a state estimation procedure in the discrete controlled Markov Chain approximation, which has not been done by other researchers. Our simulation shows that this procedure does improve the control effect than that there only the delayed information is used. In order to reduce the huge memory required by this approach and better predict the present state, we present an idea to keep the control history in the state representation. In this way, we are able to find the accurate probability distribution of the present state. It also makes the optimality proof possible. This extension of controlled Markov Chain approximation to solve stochastic control problem with time delay is also new. For some long path dependent stochastic models, the Monte Carlo simulation with Euler scheme is the best approach so far. We examined the pricing of mortgage backed securities using Monte Carlo simulation. By doing the Option Adjusted Spread (OAS) analysis, we found that instead of achieving the absolute convergence, we should better use the relative convergence to calculate the duration or convexity of the bond. The computation is speed up dramatically.

Finally for a certain form of path dependence in the stochastic model, the Ito's formula and Dynkin formula have closed forms. By examining the behavior of financial traders, we presented a SDDE model describing the underlying asset, and use this model in the classical portfolio optimization problem. Due to the specific form, this problem was deduced to a PDE.